WS#11-3
Geometric Sequences and Series

1. **You will be responsible to read the section completely and review the definitions and properties of the following:**

   A. geometric sequence → The ratio of successive terms is always the same non-zero number.

   B. common ratio = r (the ratio of successive terms in a geometric sequence)

   C. recursive formula of geometric sequence → $a_1 = a$; $r = \frac{a_n}{a_{n-1}}$ $\boxed{a_n = r(a_{n-1})}$

   D. explicit formula of geometric sequence → $\boxed{a_n = a_1(r^{n-1}) \text{ where } r \neq 0}$

   E. sum of a finite number of geometric sequence → $\boxed{S_n = \frac{a_1(1-r^n)}{1-r}, \ r \neq 0,1}$

   F. sum of an infinite of geometric series → $\sum_{K=1}^{\infty} ar^{K-1} = \frac{a_1}{1-r}$

   G. Convergent series → If S_n (the sum of an infite geometric series) approaches a number L as $n \to \infty$, then we say the series converges

   $L = \sum_{K=1}^{\infty} ar^{K-1}$

   H. Divergent series → If S_n (the sum of an infinite geometric series) does not converge, it is called a divergent series

2. **Find the common ratio and first term of each of the following:**

   A. 2, 6, 18, 54, ... $a_1 =$ 2 $r =$ 3

   B. $\{s_n\} = \{2^{-n}\}$ $S_1 = \frac{1}{2}$, $S_n = 2^{-n}$, $S_{n-1} = 2^{-(n-1)}$ $a_1 = S_1 = 2^{-1} = \frac{1}{2}$ $r = \frac{S_n}{S_{n-1}} = \frac{2^{-n}}{2^{-(n-1)}} = \boxed{\frac{1}{2}}$

   • use recursive formula for a geometric sequence

3. **Find the a_n, the recursive definition and the ninth term of the sequence** $10, 9, \frac{81}{10}, \frac{729}{100}, \ldots$

   $a_1 = 10$, $r = \frac{9}{10}$

   $a_n = \frac{9}{10}(a_{n-1})$ **Recursive definition:** $a_n = r(a_{n-1}), a_1 = 10$

   $a_9 = 10\left(\frac{9}{10}\right)^{9-1} = 10\left(\frac{9}{10}\right)^8 \approx \boxed{4.305}$

4. **Find the sum of the first n terms of the sequence** $\left\{\left(\frac{1}{2}\right)^n\right\}$. $a_1 = \frac{1}{2}$, $r = \frac{1}{2}$

   $S_n = \sum_{K=1}^{\infty} \left(\frac{1}{2}\right)^K = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \left(\frac{1}{2}\right)^n \longrightarrow \frac{1}{2}\left[\frac{1-\left(\frac{1}{2}\right)^n}{1-\left(\frac{1}{2}\right)}\right] = \frac{1}{2}\left[\frac{1-\left(\frac{1}{2}\right)^n}{\frac{1}{2}}\right] = \boxed{1-\left(\frac{1}{2}\right)^n}$

5. **Find the sum of the first 15 terms of the sequence** $\left\{\left(\frac{1}{3}\right)^n\right\}$.

   $S_{15} = \sum_{K=1}^{15} \left(\frac{1}{3}\right)^K = \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots + \left(\frac{1}{3}\right)^{15} = \frac{1}{3}\left[\frac{1-\left(\frac{1}{3}\right)^{15}}{1-\left(\frac{1}{3}\right)}\right] = \frac{1}{3}\left[\frac{1-\left(\frac{1}{3}\right)^{15}}{\frac{2}{3}}\right]$

   TI 83 or 84 → sum(seq((1/3)^n, n, 1, 15, 1))

   $\approx \boxed{0.4999999652}$

6. What is the difference between a sequence and a series? A sequence is a list of terms separated by a comma, whereas a series is a list of terms separated by addition (+) signs

7. Does the geometric series $2+\frac{4}{3}+\frac{8}{9}+\dots$ converge? If it converges find the sum. Converges $\boxed{6}$

$a_1 = 2$

$r = \frac{2}{3}$

$S_n = a_1\left[\frac{1-r^n}{1-r}\right] = \frac{a}{1-r} - \frac{ar^n}{1-r}$

Since $|r|<1$, $\sum_{k=1}^{\infty} ar^{k-1} = \frac{a}{1-r} = \frac{2}{1-\frac{2}{3}}$

8. Show that the repeating decimal 0.9999999.... equals 1.

$\hookrightarrow 0.999\ldots = \frac{9}{10} + \frac{9}{100} + \frac{9}{1000} + \ldots$

• use $\frac{a}{1-r} = \frac{\frac{9}{10}}{1-(\frac{1}{10})} = \boxed{1}$

$0.999\ldots$ is a geometric series $\rightarrow a_1 = \frac{9}{10}$, $r = \frac{1}{10}$

9. Initially, a pendulum swings through an arc of 18 inches. On each successive swing, the length of the arc is 0.98 of the previous length.

A. What is the length of the arc of the 10th swing?

B. On which swing is the length of the arc first less than 12 inches?

C. After 15 swings, what is the total distance the pendulum has swung?

D. When it stops, what total distance will the pendulum have swung?

A.) 1st swing = 18 in, 2nd swing = $(.98)(18)$ in, 3rd swing = $.98(.98)18 = (.98)^2(18)$

10th swing = $(.98)^9(18) \approx \boxed{15.007 \text{ in}}$

B.) The length of the arc of the nth swing is $(0.98)^{n-1}(18)$

$\hookrightarrow \frac{12}{18} = \frac{(0.98)^{n-1}(18)}{18} \rightarrow \frac{2}{3} = (0.98)^{n-1} \rightarrow \ln\left(\frac{2}{3}\right) = \ln(0.98)^{n-1}$

$\frac{\ln\left(\frac{2}{3}\right)}{\ln(0.98)} = \frac{(n-1)\cancel{\ln(0.98)}}{\cancel{\ln(0.98)}} \rightarrow \frac{\ln\left(\frac{2}{3}\right)}{\ln(0.98)} + 1 = n \rightarrow \boxed{21.07 = n}$

* The length of the arc of the pendulum exceeds 12 in on the 21st swing and is less than 12 in on the $\boxed{\text{22nd swing}}$.

C.) $L = \underset{(1st)}{18} + \underset{(2nd)}{0.98(18)} + \underset{(3rd)}{(0.98)^2(18)} + \underset{(4th)}{(0.98)^3(18)} + \ldots + \underset{(15th)}{(0.98)^{14}(18)}$

$a_1 = 18$, $r = 0.98 \rightarrow L = a_1\left[\frac{1-r^n}{1-r}\right] = 18\left[\frac{1-(0.98)^{15}}{1-0.98}\right] \approx \boxed{235.3 \text{ in}}$

D.) Total Distance $= 18 + (0.98)(18) + (0.98)^2(18) + \ldots$

This is the sum of an infinite geometric series

Total Distance $= \frac{a}{1-r} = \frac{18}{1-0.98} = \underline{900}$

• The pendulum will have swung 900 in. when it finally stops.